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# Logarithmic correction to the Brane equation in topological Reissner–Nordström de Sitter space

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**Abstract.** In this paper we study braneworld cosmology when the bulk space is a charged black hole in de Sitter space (topological Reissner–Nordström de Sitter Space) in a general number of dimensions; then we compute the leading order correction to the Friedmann equation that arises from logarithmic corrections to the entropy in the holographic-braneworld cosmological framework. Finally we consider the holographic entropy bounds in this scenario, and we show that the entropy bounds are also modified by a logarithmic term.

## 1 Introduction

Holography is believed to be one of the fundamental principles of the true quantum theory of gravity [1,2]. An explicitly calculable example of holography is the much-studied AdS/CFT correspondence [3]. Unfortunately, it seems that we live in a universe with a positive cosmological constant which will look like de Sitter space-time in the far future. Therefore, we should try to understand quantum gravity or string theory in de Sitter space preferably in a holographic way. Of course, physics in de Sitter space is interesting even without its connection to the real world; de Sitter entropy and temperature have always been mysterious aspects of quantum gravity [4].

While string theory successfully has addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional [5,6]. Another, related, reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences is much more tricky and has not been given a satisfactory answer within string theory. While the idea of black hole complementarity provides useful clues [7], rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

More recently, it has been proposed that, defined in a manner analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [8] (see also earlier works [9–12]). Following this proposal, some investigations on the dS space have been carried out recently [10–28]. According to the dS/CFT correspondence, it might be expected that, as in the case of AdS black holes [29], the thermodynamics of the cosmological horizon in asymptotically dS spaces can be identified with that of a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces.

There has been much recent interest in calculating the quantum corrections to $S_{\rm BH}$ (the Bekenstein–Hawking entropy) [30–46]. The leading-order correction is proportional to $\ln S_{\rm BH}$. There are *two* distinct and separable sources for this logarithmic correction [44,45] (see also the recent paper by Gour and Medved [46]). Firstly, there should be a correction to the number of microstates, that is, a quantum correction to the microcanonical entropy, and secondly, as any black hole will typically exchange heat or matter with its surrounding, there should also be a correction due to thermal fluctuations in the horizon area.

In this paper we consider the brane universe in the bulk background of the topological Reissner–Nordström de Sitter (TRNdS) black holes. In fact there is pressing cosmological motivation for introducing the CFT potential dual to the charge of the black hole. It is, in particular, the presence of a non-vanishing charge that can induce the desirable feature of a non-vanishing bounce [47]. First we find the thermodynamical quantities of the dual CFT, and then we show that the Friedmann brane equation can be in the Cardy–Verlinde formula when the brane crosses the black hole horizon or the cosmological horizon. Taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole horizon entropies. As a result the Cardy–Verlinde formula and Friedmann brane equation receive logarithmic corrections.

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Therefore, here we generalize the logarithmic corrections (with respect to the temperature) that appear in the five-dimensional case of [43, 48] to any dimension, but in the specific case of a topological Reissner–Nordström black hole in de Sitter (dS) space. The arguments are based on dS/CFT correspondence for the counting of microstates, which of course is not a well established result. However, one may accept this conjecture and study its consequences. Finally we consider the implication of the prior analysis with regard to the holographic entropy bounds, and we show that the entropy bounds are also modified by a logarithmic term.

## 2 FRW equation in the background of TRNdS black holes

The topological Reissner–Nordström dS black hole solution in $(n+2)$-dimensions has the following form:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\,,$$
$$f(r) = k - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2}\,, \qquad (1)$$

where

$$\omega_n = \frac{16\pi G}{n\mathrm{Vol}(\Sigma)}, \qquad \phi = -\frac{n}{4(n-1)}\frac{\omega_n Q}{r^{n-1}}\,, \qquad (2)$$

where $Q$ is the electric/magnetic charge of the Maxwell field, $M$ is assumed to be a positive constant, $l$ is the curvature radius of de Sitter space, $\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$ denotes the line element of an $n$-dimensional hypersurface $\Sigma_k$ with the constant curvature $n(n-1)k$, and its volume is $V(\Sigma_k)$. $\Sigma_k$ is in the spherical ($k=1$), flat ($k=0$), or hyperbolic ($k=-1$) case, and $\phi$ is the electrostatic potential related to the charge $Q$. When $k=1$, the metric (1) is just the Reissner–Nordström–de Sitter solution. For general $M$ and $Q$, the equation $f(r)=0$ may have four real roots. Three of them are real, the largest one is the cosmological horizon $r_{\mathrm{c}}$, the smallest is the inner (Cauchy) horizon of the black hole, the middle one is the outer horizon $r_+$ of the black hole. The fourth is negative and has no physical meaning. The case $M=Q=0$ reduces to the de Sitter space with a cosmological horizon $r_{\mathrm{c}}=l$.

When $k=0$ or $k=-1$, there is only one positive real root of $f(r)$, and this locates the position of the cosmological horizon $r_{\mathrm{c}}$.

In the case of $k=0$, $\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$ is an $n$-dimensional Ricci flat hypersurface; when $M=Q=0$ the solution (1) goes to a pure de Sitter space

$$\mathrm{d}s^2 = \frac{r^2}{l^2}\mathrm{d}t^2 - \frac{l^2}{r^2}\mathrm{d}r^2 + r^2\mathrm{d}x_n^2\,, \qquad (3)$$

in which $r$ becomes a timelike coordinate.

When $Q=0$, and $M\to -M$ the metric (1) is the TdS (topological de Sitter) solution [21], which has a cosmological horizon and a naked singularity.

For the purpose of getting the Friedmann–Robertson–Walker (FRW) metric, we impose the following condition [23]:

$$\frac{1}{f(r)}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 - f(r)\left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^2 = -1\,, \qquad (4)$$

which leads to a timelike brane. Substituting (4) into the TRNdS solution (1), one has the induced brane metric which takes the FRW form:

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + r^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\,. \qquad (5)$$

A timelike brane, i.e. a brane that has a Minkowskian metric, can only cross the black hole horizon. On the contrary, a spacelike brane, i.e. a brane with Euclidean metric, is able to cross both the black hole horizon and the cosmological horizon. In order to derive the 4-dimensional spacelike brane, the imposed condition (4) has to be slightly changed by replacing the "−" with a "+" on the right-hand side of it.

The equation of motion of the brane is given by [24]

$$\mathcal{K}_{ij} = \frac{\sigma}{n}h_{ij}\,, \qquad (6)$$

where $\mathcal{K}_{ij}$ is the extrinsic curvature, and $h_{ij}$ is the induced metric on the brane; $\sigma$ is the brane tension. The extrinsic curvature, $\mathcal{K}_{ij}$, of the brane can be calculated and expressed in terms of the function $r(\tau)$ and $t(\tau)$. Thus one rewrites the equations of motion (6) as follows:

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{\sigma r}{f(r)}\,. \qquad (7)$$

Using (4) and (7), we can derive the FRW equation with $H = \frac{\dot{r}}{r}$,

$$H^2 = \frac{f(r)}{r^2} + \sigma^2 = \frac{-\omega_n M}{r^{n+1}} + \frac{nw_n^2Q^2}{8(n-1)r^{2n}} + \frac{k}{r^2} - \frac{1}{l^2} + \sigma^2\,, \qquad (8)$$

where $H$ is the Hubble parameter. We choose the brane tension $\sigma = \frac{1}{l}$ to obtain a critical brane. Therefore (8) leads to

$$H^2 = \frac{-\omega_n M}{r^{n+1}} + \frac{nw_n^2Q^2}{8(n-1)r^{2n}} + \frac{k}{r^2}\,. \qquad (9)$$

Making use of the fact that the metric for the boundary CFT can be determined only up to a conformal factor, we rescale the boundary metric for the CFT to be of the following form:

$$\mathrm{d}s^2_{\mathrm{CFT}} = \lim_{r\to\infty}\left[\frac{l^2}{r^2}\mathrm{d}s^2_{n+2}\right] = \mathrm{d}t^2 + l^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\,. \qquad (10)$$

Evidently, the Euclidean CFT time must be scaled by a factor $l/r$. Proceeding on this basis, the thermodynamic relations between the boundary CFT and the bulk TRNdS are given by

$$E_{\mathrm{CFT}} = M\frac{l}{r}\,, \quad \Phi_{\mathrm{CFT}} = \Phi\frac{l}{r}\,,$$

$$T_{\mathrm{CFT}} = T_{\mathrm{TRNdS}} \frac{l}{r}\,, \quad S_{\mathrm{CFT}} = S_{\mathrm{TRNdS}}\,, \tag{11}$$

where the black hole horizon Hawking temperature $T^{\mathrm{b}}_{\mathrm{TRNdS}}$ and entropy $S^{\mathrm{b}}_{\mathrm{TRNdS}}$ are given by

$$T^{\mathrm{b}}_{\mathrm{TRNdS}} = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left((n-1) - (n+1)\frac{r_+^2}{l^2} - \frac{n\omega_n^2 Q^2}{8r_+^{2n-2}}\right),$$

$$S^{\mathrm{b}}_{\mathrm{TRNdS}} = \frac{r_+^n \mathrm{Vol}(\Sigma)}{4G}\,, \tag{12}$$

where $r = r_+$ is the black hole horizon and $V_+ = r_+^n \mathrm{Vol}(\Sigma)$ is the area of it in $(n+2)$-dimensional asymptotically dS space.

Here we review the BBM prescription [17] for computing the conserved quantities of asymptotically de Sitter spacetimes briefly. In a theory of gravity, mass is a measure of how much a metric deviates near infinity from its natural vacuum behavior; i.e., mass measures the warping of space. Inspired by the analogous reasoning in AdS space [49, 50] one can construct a divergence-free Euclidean quasilocal stress tensor in de Sitter space by the response of the action to variation of the boundary metric, and we have

$$T^{\mu\nu} = \frac{2}{\sqrt{h}}\frac{\delta I}{\delta h_{\mu\nu}} = \frac{1}{8\pi G}\left[K^{\mu\nu} - K\,h^{\mu\nu} + \frac{n}{l}\,h^{\mu\nu} + \frac{l}{n}\,G^{\mu\nu}\right], \tag{13}$$

where $h^{\mu\nu}$ is the metric induced on surfaces of fixed time, $K_{\mu\nu}$, $K$ are respectively the extrinsic curvature and its trace, and $G^{\mu\nu}$ is the Einstein tensor of the boundary geometry. To compute the mass and other conserved quantities, one can write the metric $h^{\mu\nu}$ in the following form:

$$h_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = N_\rho^2\,\mathrm{d}\rho^2 + \sigma_{ab}\,(\mathrm{d}\phi^a + N^a_\Sigma\,\mathrm{d}\rho)\,(\mathrm{d}\phi^b + N^b_\Sigma\,\mathrm{d}\rho)\,, \tag{14}$$

where the $\phi^a$ are angular variables parameterizing closed surfaces around the origin. When there is a Killing vector field $\xi^\mu$ on the boundary, then the conserved charge associated to $\xi^\mu$ can be written as [49, 50]

$$Q = \oint_\Sigma \mathrm{d}^n\phi\,\sqrt{\sigma}\,n^\mu\xi^\mu\,T_{\mu\nu}\,, \tag{15}$$

where $n^\mu$ is the unit normal vector on the boundary, $\sigma$ is the determinant of the metric $\sigma_{ab}$. Therefore the mass of an asymptotically de Sitter space is

$$M = \oint_\Sigma \mathrm{d}^n\phi\,\sqrt{\sigma}\,N_\rho\,\epsilon; \quad \epsilon \equiv n^\mu n^\nu\,T_{\mu\nu}\,, \tag{16}$$

where the Killing vector is normalized by $\xi^\mu = N_\rho n^\mu$. Using this prescription [17], the gravitational mass, subtracting the anomalous Casimir energy, of the TRNdS solution is

$$E^{\mathrm{c}} = -M = -\frac{r_{\mathrm{c}}^{n-1}}{\omega_n}\left(k - \frac{r_{\mathrm{c}}^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_{\mathrm{c}}^{2n-2}}\right). \tag{17}$$

The Hawking temperature $T^{\mathrm{c}}_{\mathrm{TRNdS}}$ and entropy $S^{\mathrm{c}}_{\mathrm{TRNdS}}$ associated with the cosmological horizon are

$$T^{\mathrm{c}}_{\mathrm{TRNdS}} = \frac{-f'(r_{\mathrm{c}})}{4\pi} = \frac{1}{4\pi r_{\mathrm{c}}}\left(-(n-1)k + (n+1)\frac{r_{\mathrm{c}}^2}{l^2} + \frac{n\omega_n^2 Q^2}{8r_{\mathrm{c}}^{2n-2}}\right),$$

$$S^{\mathrm{c}}_{\mathrm{TRNdS}} = \frac{r_{\mathrm{c}}^n \mathrm{Vol}(\Sigma)}{4G}\,, \tag{18}$$

where $V_{\mathrm{c}} = r_{\mathrm{c}}^n \mathrm{Vol}(\Sigma)$ is the area of the cosmological horizon. The AD mass of the TRNdS solution can be expressed in terms of the black hole horizon radius $r_+$ and charge $Q$,

$$E^{\mathrm{b}} = M = \frac{r_+^{n-1}}{\omega_n}\left(1 - \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_+^{2n-2}}\right). \tag{19}$$

In terms of the energy density $\rho_{\mathrm{CFT}} = E_{\mathrm{CFT}}/V$, the pressure $p_{\mathrm{CFT}} = \rho_{\mathrm{CFT}}/n$, the charge density $\rho_{\mathrm{QCFT}} = Q/V$ and the electrostatic potential $\Phi_{\mathrm{CFT}} = \Phi\frac{l}{r}$ of the CFT within the volume $V = r^n \mathrm{Vol}(\Sigma)$, also the specific heat of the black hole is given by

$$C^{\mathrm{c,b}} = \frac{\mathrm{d}E^{\mathrm{c,b}}}{\mathrm{d}T} \tag{20}$$
$$= \frac{4\pi r_{\mathrm{c},+}^2(8k(1-n)l^2 r_{\mathrm{c},+}^{n-2} + 8(n+1)r_{\mathrm{c},+}^n + n\omega_n^2 l^2 r_{\mathrm{c},+}^{-n} Q^2)}{\omega_n(8l^2(n-1)k + 8r_{\mathrm{c},+}^2(n+1) + (1-2n)l^2\omega_n^2 r_{\mathrm{c},+}^{2-2n} Q^2)}.$$

As one can see the above specific heat is positive in the case $k=-1, k=0$, for $k=1$, and $C^{\mathrm{c,b}}$ is positive only with the following condition:

$$8(n+1)r_{\mathrm{c},+}^n + n\omega_n^2 l^2 r_{\mathrm{c},+}^{-n} Q^2 > 8k(1-n)l^2 r_{\mathrm{c},+}^{n-2} \tag{21}$$

The first Friedmann equation takes the following form:

$$H^2 = \frac{16\pi G}{n(1-n)}\left(\rho_{\mathrm{CFT}} - \frac{1}{2}\Phi\rho_{\mathrm{QCFT}}\right) + \frac{k}{r^2}\,. \tag{22}$$

## 3 Logarithmic correction to the Cardy–Verlinde formula and FRW brane cosmology in TRNdS bulk

There has been much recent interest in calculating the quantum corrections to $S_{\mathrm{BH}}$ (the Bekenstein–Hawking entropy) [30–53]. The corrected formula takes the form

$$\mathcal{S} = S_0 - \frac{1}{2}\ln C + \ldots \tag{23}$$

When $r_{\mathrm{c},+}^2 \gg l^2$, $C \simeq nS_0$, we have

$$\mathcal{S} = S_0 - \frac{1}{2}\ln S_0 + \ldots \tag{24}$$

It is now possible to derive the corresponding correction to the Cardy–Verlinde formula. The Casimir energy $E_{\mathrm{C}}$, defined as

$$E_{\mathrm{C}}^{\mathrm{c,b}} = (n+1)E^{\mathrm{c,b}} - nT^{\mathrm{c,b}}S^{\mathrm{c,b}} - n\phi^{\mathrm{c,b}}Q\,, \tag{25}$$

in this case is found to be

$$E_{\mathrm{C}}^{\mathrm{c,b}} = \frac{-2nkr_{\mathrm{c},+}^{n-1}\mathrm{Vol}(\Sigma)}{16\pi G}\,, \tag{26}$$

which is valid for both cosmological and black hole horizons. One can see that the entropy (18) and (12) of the cosmological and black hole horizon can be written as

$$S^{\mathrm{c,b}} = \frac{2\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}^{\mathrm{c,b}}}{k}\right|(2(E^{\mathrm{c,b}} - E_q^{\mathrm{c,b}}) - E_{\mathrm{C}}^{\mathrm{c,b}})}\,, \tag{27}$$

where

$$E_q^{\mathrm{c,b}} = \frac{1}{2}\phi^{\mathrm{c,b}}Q = -\frac{n}{8(n-1)}\frac{\omega_n Q^2}{r_{\mathrm{c},+}^{n-1}}\,. \tag{28}$$

For the present discussion, the total entropy is assumed to be of the form (24), where the uncorrected entropy, $S_0$, is in correspondence to the associated one in (18) and (12). It then follows by employing (12)– (19) that the Casimir energy (25) can be expressed in terms of the uncorrected entropy. (The following expressions are valid for both cosmological and black hole horizon; for simplicity we omit the subscript c and b.) We have

$$E_{\mathrm{C}} = \frac{-2nr_{\mathrm{c},+}^{n-1}\mathrm{Vol}(\Sigma)}{16\pi G} + \frac{nT}{2}\mathrm{Ln}S_0\,, \tag{29}$$

After some calculation, the total entropy (24), to first order in the logarithmic term, is given by [51]

$$\begin{aligned} S \simeq{}& \frac{2\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})} \\ &+ \{E_q[(3n+1)E - 2nE_q + (1-2n)E_{\mathrm{C}}] \\ &\quad + E[nE_{\mathrm{C}} - (n+1)E]\} \\ &\times \frac{1}{4E_{\mathrm{C}}(E - E_q - E_{\mathrm{C}}/2)} \\ &\times \mathrm{Ln}\left(\frac{2\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})}\right)\,. \end{aligned} \tag{30}$$

Therefore taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole entropies. As a result the Cardy–Verlinde formula receives logarithmic corrections in our TRNdS black hole background of interest in any number of dimensions.

The first Friedmann equation (22) can be rewritten in terms of thermodynamical formulas of the CFT on the brane when the brane crosses the cosmological or event horizon [24,26], at these times the first Friedmann equation coincides with the Cardy–Verlinde formula. As a direct consequence of the logarithmic correction arising in (30) the Friedmann equation also receives a logarithmic correction due to thermal fluctuations of the bulk gravity system. The Hubble parameter $H$ is related with the Hubble entropy by

$$S_{\mathrm{H}} \equiv (n-1)\frac{HV}{4G}\,, \tag{31}$$

which is equal to the bulk black hole entropy at the moment when the brane crosses the black hole horizon $r = r_+$ in the case $k = 1$, and crosses the cosmological horizon $r = r_{\mathrm{c}}$ for the cases $k = 0$ and $k = -1$ [24,26]. By substituting (30) into (31) one finds the modified Friedmann equation at the holographic points:

$$\begin{aligned} H^2 ={}& \frac{16G^2}{(n-1)^2V^2}S^2 \\ ={}& \frac{16G^2}{(n-1)^2V^2}\left[\left(\frac{2\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})}\right)^2\right. \\ &+ \frac{4\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})} \\ &\times \{E_q[(3n+1)E - 2nE_q + (1-2n)E_{\mathrm{C}}] \\ &\quad + E[nE_{\mathrm{C}} - (n+1)E]\} \\ &\times \frac{1}{4E_{\mathrm{C}}(E - E_q - E_{\mathrm{C}}/2)} \\ &\left.\times \mathrm{Ln}\left(\frac{2\pi l}{n}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})}\right)\right]\,; \end{aligned} \tag{32}$$

after setting $E_q = 0, n = 3, k = 1$ the above equation agrees with the result of [48] for the Friedmann brane equation in 5-dimensional Schwarzschild de Sitter bulk which is as follows:

$$\begin{aligned} H^2 ={}& \left(\frac{2G}{V}\right)^2\left[\left(\frac{4\pi l}{3\sqrt{2}}\right)^2\left|E_{\mathrm{C}}\left(E - \frac{1}{2}E_{\mathrm{C}}\right)\right|\right. \\ &- \frac{4\pi l}{3\sqrt{2}}\frac{E\,(4E - 3E_{\mathrm{C}})}{(2E - E_{\mathrm{C}})\,E_{\mathrm{C}}} \\ &\left.\times\sqrt{\left|E_{\mathrm{C}}\left(E - \frac{1}{2}E_{\mathrm{C}}\right)\right|}\ln\left(\frac{4\pi l}{3\sqrt{2}}\sqrt{\left|E_{\mathrm{C}}\left(E - \frac{1}{2}E_{\mathrm{C}}\right)\right|}\right)\right]\,. \end{aligned} \tag{33}$$

At the holographic points $r = r_{\mathrm{c},+}$, after setting $\sigma = \frac{1}{l}$, we have

$$H^2 = \frac{1}{l^2} \quad \text{at} \quad r = r_{+,c}\,. \tag{34}$$

Formally, the Friedmann equation (32) holds precisely at the instant when the brane crosses the black hole and cosmological horizons. Here we extend the analysis to a consideration of an arbitrary scale factor $r$ where the world-volume of the brane is given by the line element (5). Thus, around each of the horizons we assume the Friedmann equation as follows:

$$\begin{aligned} H^2 ={}& \frac{k}{r^2} + \frac{16\pi G}{n(1-n)}\left(\rho_{\mathrm{CFT}} - \frac{1}{2}\Phi\rho_{\mathrm{QCFT}}\right) \\ &+ \frac{32\pi G^2 l}{(n-1)^2nV^2}\sqrt{\left|\frac{E_{\mathrm{C}}}{k}\right|(2(E - E_q) - E_{\mathrm{C}})} \\ &\times \{E_q[(3n+1)E - 2nE_q + (1-2n)E_{\mathrm{C}}] \end{aligned}$$

$$+E[nE_{\rm C}-(n+1)E]\}$$
$$\times\frac{1}{4E_{\rm C}(E-E_q-E_{\rm C}/2)}$$
$$\times{\rm Ln}\left(\frac{2\pi l}{n}\sqrt{\left|\frac{E_{\rm C}}{k}\right|(2(E-E_q)-E_{\rm C})}\right)\,; \qquad (35)$$

then the logarithmic corrections for the FRW equation are given by the last term on the right-hand side in terms of the uncorrected entropy (12) and (18) of the black hole. Here the logarithmic corrections have been included up to first order in the logarithmic term. Therefore, at least the brane receives thermal radiation from the black hole, and the thermal correction should change the dynamics of the brane from the leading order or zero temperature behavior.

Verlinde pointed out that the FRW equation (9) can be related to three cosmological entropy bounds:

$$S_{\rm BH}=(n-1)\frac{V}{4GR}$$
$$\text{(Bekenstein–Hawking bound)}, \qquad (36)$$
$$S_{\rm BV}=-\frac{2\pi R}{n}\left(E-\frac{2\pi G_nQ^2}{(n-1)V}\right)$$
$$\text{(Bekenstein–Verlinde bound)}, \qquad (37)$$

and the Hubble bound which is given by (31). Here $G_n$ is the gravitational constant in bulk which is given by

$$G_n=\frac{Gl}{n-1}\,. \qquad (38)$$

The FRW equation (9) can be rewritten as

$$S_{\rm H}=\sqrt{S_{\rm BH}(S_{\rm BH}-2S_{\rm BV})}\,, \qquad (39)$$

and similarly we can rewrite the modified Friedmann (35) as

$$S_{\rm H}=\sqrt{S_{\rm BH}(kS_{\rm BH}-2S_{\rm BV})+AS_{\rm c}\ln(S_{\rm c})}\,, \qquad (40)$$

where

$$S_{\rm c}=\frac{2\pi l}{n}\sqrt{\left|\frac{E_{\rm c}}{k}\right|(2(E-Eq)-E_{\rm c})}\,, \qquad (41)$$
$$A=\{E_q[(3n+1)E-2nE_q+(1-2n)E_{\rm c}]$$
$$+E[nE_{\rm c}-(n+1)E]\}$$
$$\times\frac{1}{4E_{\rm c}(E-E_q-E_{\rm c}/2)}\,. \qquad (42)$$

Now if we consider $k=1$ and also conjecture the redefined Bekenstein–Hawking entropy to be

$$S_{\rm BH}\to S'_{\rm BH}=S_{\rm BH}-\frac{AS_{\rm c}\ln S_{\rm c}}{2(S_{\rm BV}-S_{\rm BH})}\,, \qquad (43)$$

then (40) can be rewritten as follows:

$$S_{\rm H}=\sqrt{S'_{\rm BH}(S'_{\rm BH}-2S_{\rm BV})}\,; \qquad (44)$$

therefore the entropy bounds are also modified by a logarithmic term.

# 4 Conclusion

One of the striking results for the dynamic dS/CFT correspondence is that the Cardy–Verlinde formula on the CFT-side coincides with the Friedmann equation in cosmology when the brane crosses the cosmological or event horizon $r=r_{{\rm c},+}$ of the topological Reissner–Nordström black hole. This means that the Friedmann equation knows the thermodynamics of the CFT. (Since conformal symmetry in the bulk is broken by the presence of a black hole, a prospectively dual boundary theory is, strictly speaking, not necessarily a conformal one. Nonetheless, for convenience sake, we continue to refer to the relevant boundary theories as CFT.) There is pressing cosmological motivation for introducing the CFT potential dual to the charge of the black hole. Such models are of significant interest because they allow for the possibility of a non-singular bounce (as opposed to a big bang/crunch) [47, 54].

For a large class of black holes, the Bekenstein–Hawking entropy formula receives additive logarithmic corrections due to thermal fluctuations. On the basis of general thermodynamic arguments, Das et al. [32] deduced that the black hole entropy can be expressed as

$$\mathcal{S}=\ln\rho=S_0-\frac{1}{2}\ln\left(C\,T^2\right)+\dots \qquad (45)$$

In this paper we have analyzed this correction of the entropy of TRNdS black hole in any number of dimensions in the light of dS/CFT. We have obtained the logarithmic correction to both cosmological and black hole entropies. Then using the form of the logarithmic correction (24) one can show the corresponding correction to the Cardy–Verlinde formula which relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimension. As a direct consequence of the logarithmic correction arising in (30) the Friedmann equation also receives a logarithmic correction due to thermal fluctuations of the bulk gravity system. Moreover, we have considered the holographic entropy bounds in this scenario, and we have shown that the entropy bounds are also modified by a logarithmic term.

It should be mentioned that in standard cosmology, where there are no corrections, the first term in right-hand side of (35) represent the curvature contribution to the brane motion. The second term can be regarded as the contribution from the radiation and it redshifts as $r^{-4}$ for a brane moving in the 5-dimensional TRNdS bulk background. The last term in the right-hand side of (35) goes like $r^{-6}$; it is dominant at early times of the brane evolution while at late times the second term, i.e. the radiative matter term, dominates and thus the last term can be neglected. At this point a couple of questions are raised. First, how does the additional term in the Hubble equation (35), which comes from thermal fluctuations, change the dynamics of the brane? The second question arises when one includes both semiclassical (the self-gravitational effect [55]) and logarithmic corrections. Which is the dominant correction and when does this dominance take place during the brane evolution? We hope to address these interesting issues in a future work.

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